Abstract

Title: Braids and the inclusion of the configuration space into the product for Surfaces and Spherical spaces.

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Given a space X we denote by $F_n(X)$ the nth configuration space of X, which is the subset of consisting of the elements $(x_1, \ldots, x_n)$ of the Cartesian product $X \times \cdots \times X$ (n copies) for which $x_i \neq x_j$ for $i \neq j$. In order to understand better $F_n(X)$, we compare certain properties of the two spaces $F_n(X)$ and $X \times \cdots \times X$ (n copies), such as their fundamental group and their homotopy type. In the latter case, this corresponds to determining the homotopy fibre of the inclusion, i.e. a space $F(\iota)$ such that

$$F(\iota) \to F_n(X) \to X \times \cdots \times X$$

looks a fibration. The two problems are related and they may be equivalent or not, depending on X. The study of these questions were motivated by the case where X is a surface. In this talk we present the recent results for the cases where X is either the sphere, the projective plane or the quotient of an odd sphere by a finite group. The groups are determined by means of a presentation, and a few of their properties are explored. Concerning the homotopy fibre of the inclusion, the results are given in terms of the spheres, loop spaces and notoriously the *equivariant configuration spaces*, a concept introduced in [CX]. The results above contain a solution of a problem for X either S^2 or RP^2, which in the case of X a closed surface different from S^2 and RP^2 it was proposed by [Bi] and solved in [Gol]. For further results see [GGG], [GG1], [GG2]. One type of result that we get, is illustrated in the following statement: *Theorem: Let $n \geq 2$. With the above notation, the homotopy fibre I_{ι_n} of the inclusion map $\iota_n : F_n(RP^2) \to \prod_1^n RP^2$ has the homotopy type of $F_{n-1}^{Z_2}(C) \times \Omega \left(\prod_1^{n-1} S^2\right)$, or equivalently of $K(G_{n-1}, 1) \times \Omega \left(\prod_1^{n-1} S^2\right)$,* where $F_{n-1}^{Z_2}(C)$ is the orbit configuration space of the cylinder and G_{n-1} is the orbit braid group i.e. $\pi_1(F_{n-1}^{Z_2}(C))$.

References

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