COMPACTIFICATION OF UNIFORMLY CONTINUOUS MAPPINGS

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Below the notion of compactification of a uniformly continuous mapping is introduced and some of their properties are established. The notion of compactification of continuous mappings has been introduced and studied in [5, 12]. A wider study of compactification of continuous mappings has been done by Pasynkov [10] and in [11, 8, 9].

All considered uniform spaces are assumed to be separated and given in coverings terms, mappings are uniformly continuous and topological spaces are Tychonoff.

Definition 1. [8]. Let $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ be uniformly continuous mapping. A mapping $cf : (cX, c\mathcal{U}) \to (Y, \mathcal{V})$ is called *compactification* or *uniformly perfect extension* of the mapping f if the following conditions hold: 1)$X \subseteq cX$; 2) $[X]_{cX}$; 3)$cf|_X = f$; 4) cf is a uniformly perfect mapping.

For two compactifications $c_1 f : (c_1X, c_1\mathcal{U}) \to (Y, \mathcal{V})$ and $c_2 f : (c_2X, c_2\mathcal{U}) \to (Y, \mathcal{V})$ of a mapping $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$, as usually we set $c_2 f \geq c_1 f$, if there is a uniformly continuous mapping $\varphi : (c_2X, c_2\mathcal{U}) \to (c_1X, c_1\mathcal{U})$, such that $c_2 f = c_1 f \cdot \varphi$ and φ is an identity mapping on X.

The notions of uniformly perfect and complete mappings are introduced and investigated by the author in [1, 2, 3, 4].

Theorem 1. *Every uniformly continuous mapping $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ has at least one compactification ($\equiv$ of one uniformly perfect extension).*

Theorem 2. *Every uniformly continuous mapping $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ has maximal compactification ($\equiv$ a maximum uniformly perfect extension).*

Theorem 3. *Let $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ be a uniformly continuous mapping. Then the following conditions are equivalent:*

(1) *A mapping f is uniformly perfect.*
(2) *A mapping f is precompact and for any compact extension $(b_cY, b\mathcal{V}_c)$ of a uniform space $(Y, \mathcal{V})$ the mapping $b_c f$ satisfies to the condition $\beta_c f(\beta_s X \setminus X) \subseteq b_c Y \setminus Y$.*
(3) *A mapping f is precompact and the mapping $\beta_s f : (\beta_s X, \beta\mathcal{U}_s) \to (\beta_s Y, \beta\mathcal{V}_s)$ satisfies $\beta_s f(\beta_s X \setminus X) \subseteq \beta_s Y \setminus Y$.*
(4) *A mapping f is precompact and there is a compact extension $(b_cY, b\mathcal{V}_c)$ of a uniform space $(Y, \mathcal{V})$, such that for the extension $\beta_s f : (\beta_s X, \beta\mathcal{U}_s) \to (\beta_s Y, \beta\mathcal{V}_s)$ of the mapping f the inclusion $\beta_s f(\beta_s X \setminus X) \subseteq \beta_s Y \setminus Y$ holds.*

Taking this into account and assuming that U is a maximal precompact uniformity of a Tychonoff space X, then Theorem 3 implies well-known theorem of Henriksen and Isbell [7] in the form, contained in [6].

The set of all compactifications of a uniformly continuous mapping $f : (X, \mathcal{U}) \to (Y, \mathcal{V})$ will be denoted as $K(f)$. The set $K(f)$ is partially ordered by the order " $\leq$ ", which we introduced earlier. A partially ordered set $(K(f), \leq)$ is not empty (Theorem 1) and has a maximal element (Theorem 2).

We denote by $C(f)$ the set of all such uniformities U_P of a space X that, firstly $U_P \subseteq U$, and, the second, a mapping $f : (X, \mathcal{U}_c) \to (Y, \mathcal{V})$ is precompact and uniformly continuous. The set $C(f)$ is partially ordered by the inclusion " $\subseteq$ ". A partially ordered set $(C(f), \subseteq)$ is not empty and has a maximal element.

Theorem 4. *There is an isomorphism* $G : (K(f), \leq) \to (C(f), \subseteq)$ *between the partially ordered sets* $(K(f), \leq)$ *and* $(C(f), \subseteq)$.

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